Structrual Analysis I

HW #4

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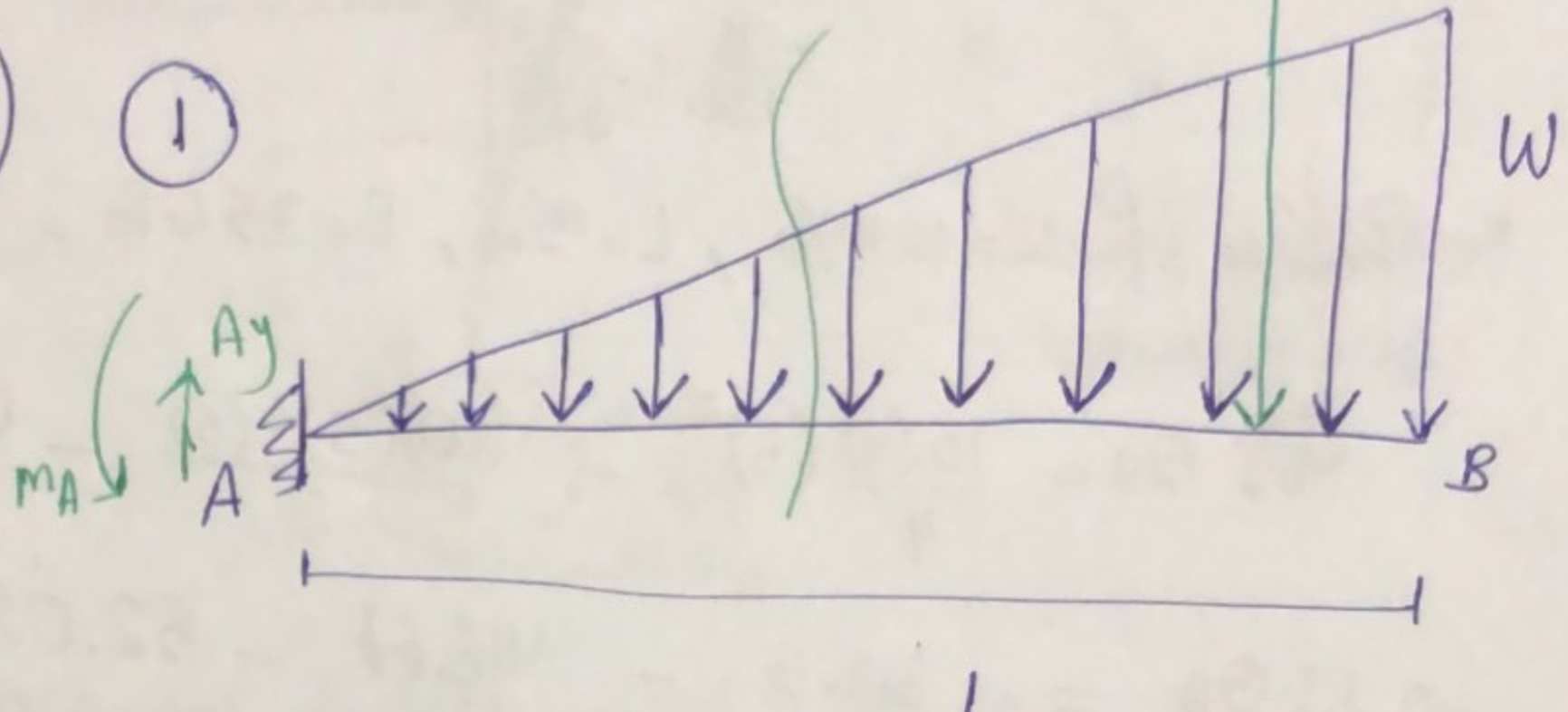
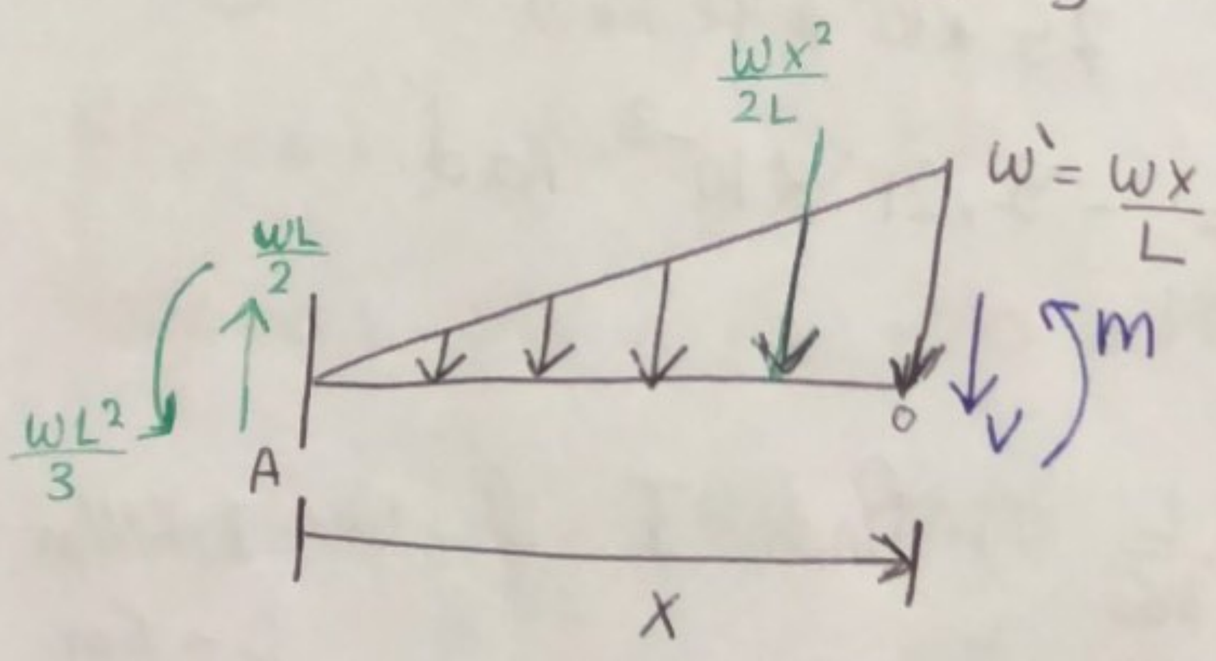
Q1:-) (1)



$$\Sigma F_y = 0: \quad A_y = \frac{WL}{2} \ kN$$

$$\Sigma M_A = 0: \quad M_A = \frac{WL}{2} \cdot \frac{2L}{3} = \frac{WL^2}{3} \ kN.m$$



$$\frac{w'}{x} = \frac{W}{L}$$

$$w' = \frac{Wx}{L}$$

$$\Sigma M_o = 0: \quad m(x) + \frac{Wx^2}{2L} \cdot \frac{x}{3} - \frac{WL}{2}x + \frac{WL^2}{3} = 0$$

$$m(x) = \frac{WLx}{2} - \frac{WL^2}{3} - \frac{Wx^3}{6L}$$

$$EI \frac{d^2V}{dx^2} = \frac{WLx}{2} - \frac{WL^2}{3} - \frac{Wx^3}{6L}$$

$$EI\theta = \frac{WL}{4}x^2 - \frac{WL^2}{3}x - \frac{Wx^4}{24L} \quad (1)$$

$$EI\,V = \frac{WL}{12}x^3 - \frac{W}{120L}x^5 - \frac{WL^2}{6}x^3$$

maximum Def. when $x = L$:

$$V(max) = \left(\frac{WL^4}{12} - \frac{WL^4}{120} - \frac{WL^4}{6}\right) / EI$$

$$= \frac{-11}{120} \cdot \frac{WL^4}{EI}$$

If $W = 10\,KN/m$, $L = 5m$, $E = 75\,GPa$, $I = 4\times10^8\,mm^4$

$$EI\,\theta_B = \frac{10(5)}{4}(5)^2 - \frac{(10)(5)^2(5)}{3} - \frac{(10)(5)^4}{24(5)}$$

$$\rightarrow EI\,\theta_B = 312.5 - 416.67 - 52.083$$

$$\rightarrow EI\,\theta_B = -156.253$$

$$\theta B = \frac{-156.253}{75 * 10^6 * 4 * 10^8 * 10^{-12}}$$

$$= -5.21 * 10^{-3}\ rad.$$

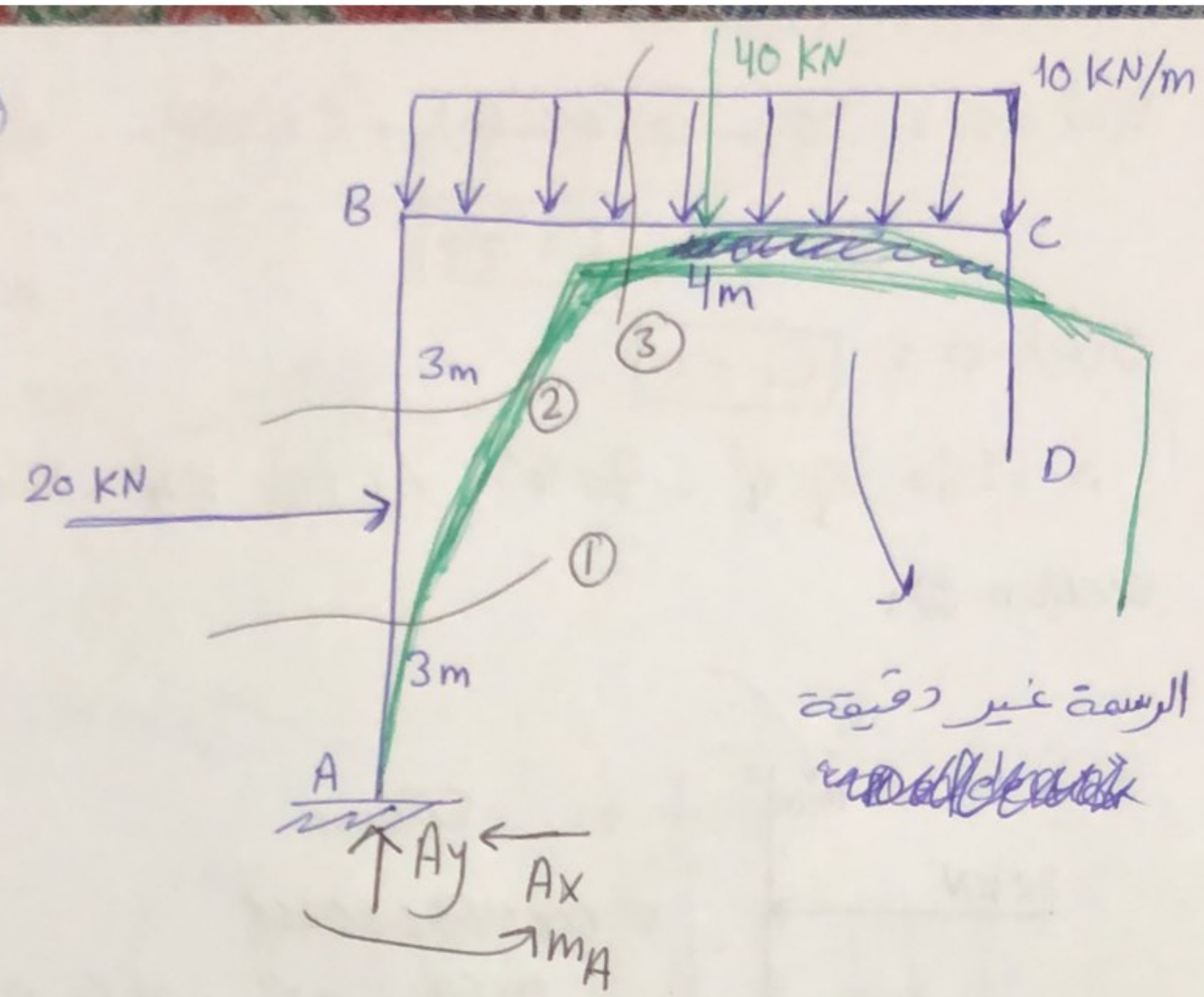
$V_{max} = \frac{L}{360}$, find I if $W = 20\,KN/m$, $L = 6m$, $E = 200\,GPa$

$$\frac{-11}{120} \cdot \frac{20(6)^3}{200 * 10^6 * I} = \frac{6}{360}$$

$$\rightarrow I = \frac{+11}{120} \cdot \frac{20.(6)^3}{200 * 10^6} \cdot 360$$

$$= +7.128 * 10^{-4}$$

Q1:) ②



الرسمة غير دقيقة

find:

ⓐ maximum horizontal displacement of the fram?

ⓑ The vertical deflection of point D?

$\Sigma F_y = 0$: $A_y = 40\,kN$

$\Sigma F_x = 0$: $A_x = 20\,kN$

$\Sigma M_A = 0$: $M_A = 60 + 80 = 140\,kN.m$

$\Sigma M_O = 0$: $M_1(x) - 20x_1 + 140 = 0$

$M_1(x) = 20x_1 - 140$

$EI\frac{d^2v}{dx^2} = 20x_1 - 140$

$EI\theta = 10x_1^2 - 140x_1 + C_1$

$EIv = \frac{10}{3}x_1^3 - 70x_1^2 + C_1x_1 + C_2$

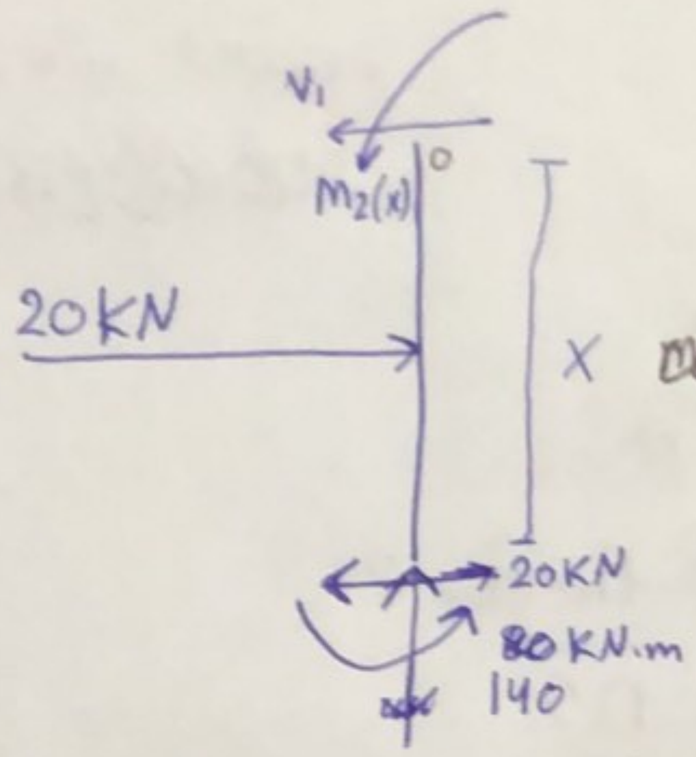
$V(0) = 0$: $0 - 0 + C_1(0) + C_2 = 0$

$\boxed{C_2 = 0}$

$\theta(0) = 0$: $\boxed{C_1 = 0}$

$\therefore EIV_1 = \frac{10}{3}x_1^3 - 70x_1^2$ / $EI\theta_1 = 10x_1^2 - 140x_1$ $[0 < x_1 < 3)$

Section ②:



$\Sigma M_O = 0$

$M_2(x) - 20X + 20(X-3) + 80 = 0$

$M_2(x) = -80$ KN.m

$EI\theta_2 = -80X + C_3$

$EIV_2 = -40X^2 + C_3X + C_4$

$\theta_1(3) = \theta_2(3)$

$10(3)^2 - 140(3) = -80(3) + C_3$

$C_3 = 90 - 420 + 240$

$\boxed{C_3 = -90}$

$V_1(3) = V_2(3)$

$\frac{10}{3}(27) - 70(9) = -40(9) + -90(3) + C_4$

$C_4 = 90 - 630 + 360 + 270$

$\boxed{C_4 = 90}$

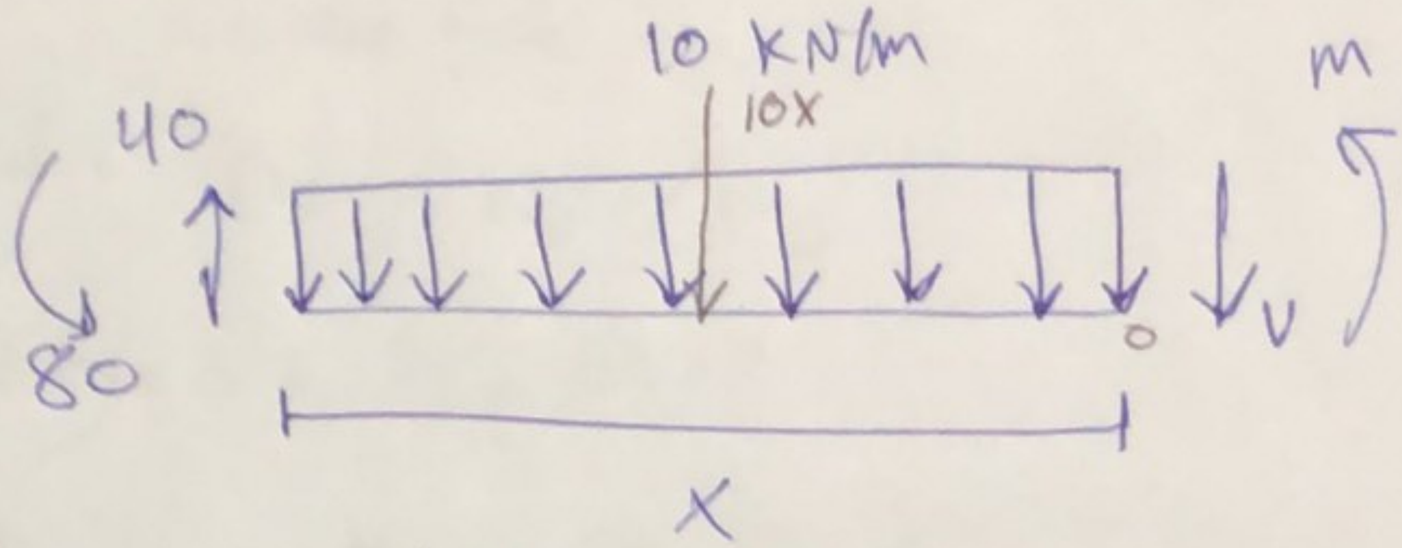
Maximum horizontal "sway" ⟶

$$\text{sway} = \frac{-40(6)^2 + (-90)(6) + 90}{EI}$$

or

$$\rightarrow \text{sway} = \frac{-1890}{EI}$$

section (3):



$$\Sigma M_o = 0 : \quad M(x) + \frac{10x^2}{2} - 40x + 80 = 0$$

$$M(x) = -5x^2 + 40x - 80$$

$$EI\theta_3 = \int M(x)\,dx$$

$$= \frac{-5}{3}x^3 + 20x^2 - 80x + C_5$$

$$EIv_3 = \frac{-5}{12}x^4 + \frac{20}{3}x^3 - 40x^2 + C_5x + C_6$$

$$v_3(0) = 0 : \quad C_6 = 0$$

$$\theta_2(6) = \theta_3(0)$$

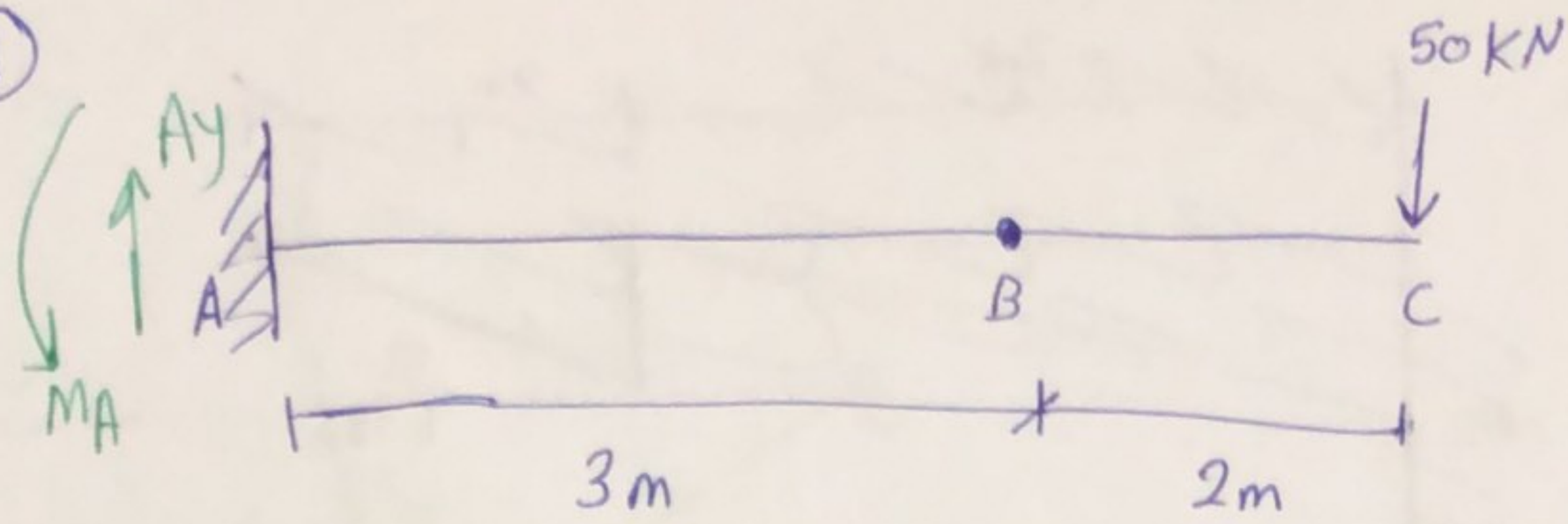
$$-80(6) - 90 = C_6 \longrightarrow C_6 = -570$$

$$EIv_D = \frac{-5}{12}(4)^4 + \frac{20}{3}(4)^3 - 40(4)^2 + (-570)(4)$$

$$v_D = -\frac{2600}{EI}\ m$$

(5)

Q2:-) ①



$I_{AB} = 5 * 10^8 \ mm^4$

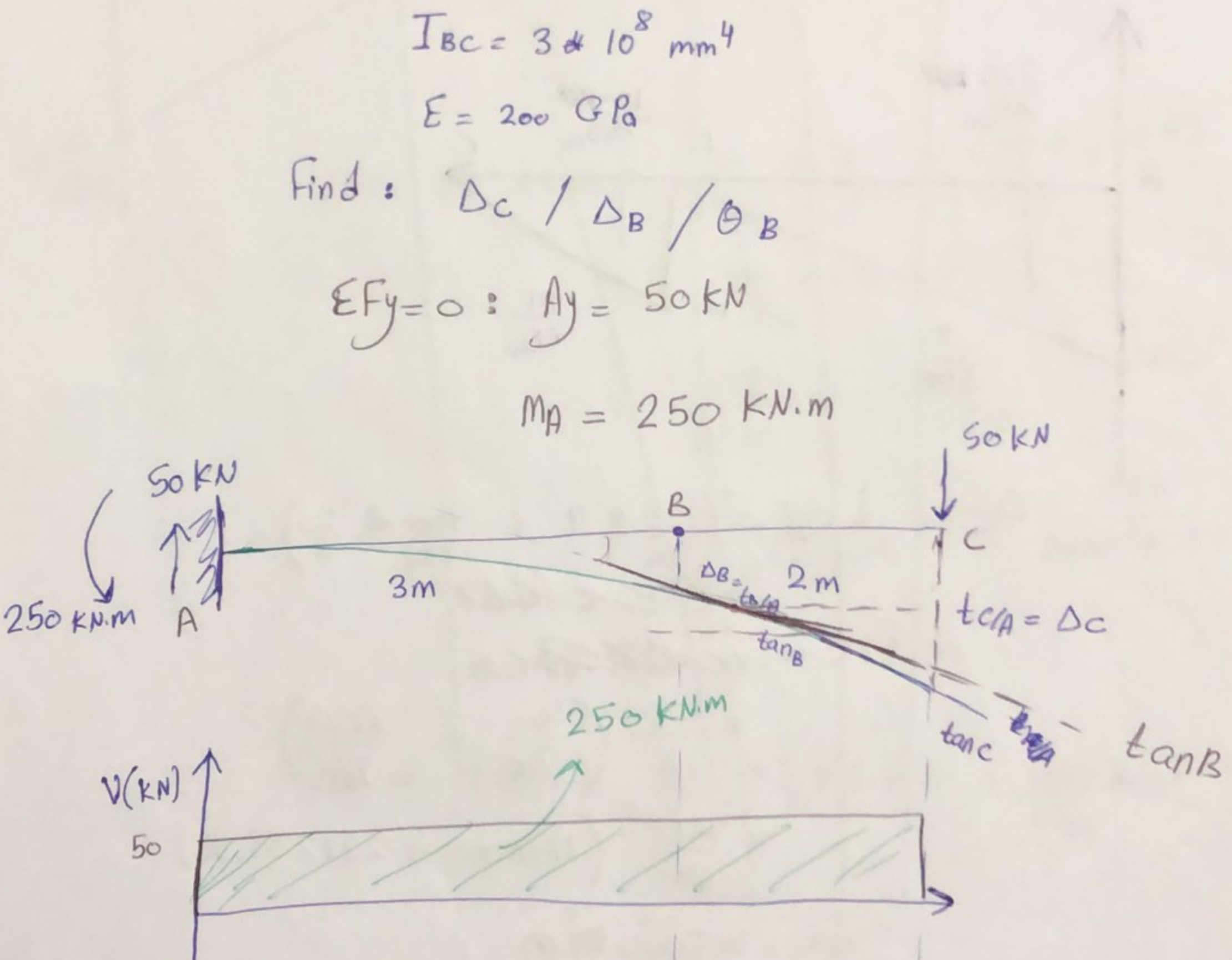
$I_{BC} = 3 * 10^8 \ mm^4$

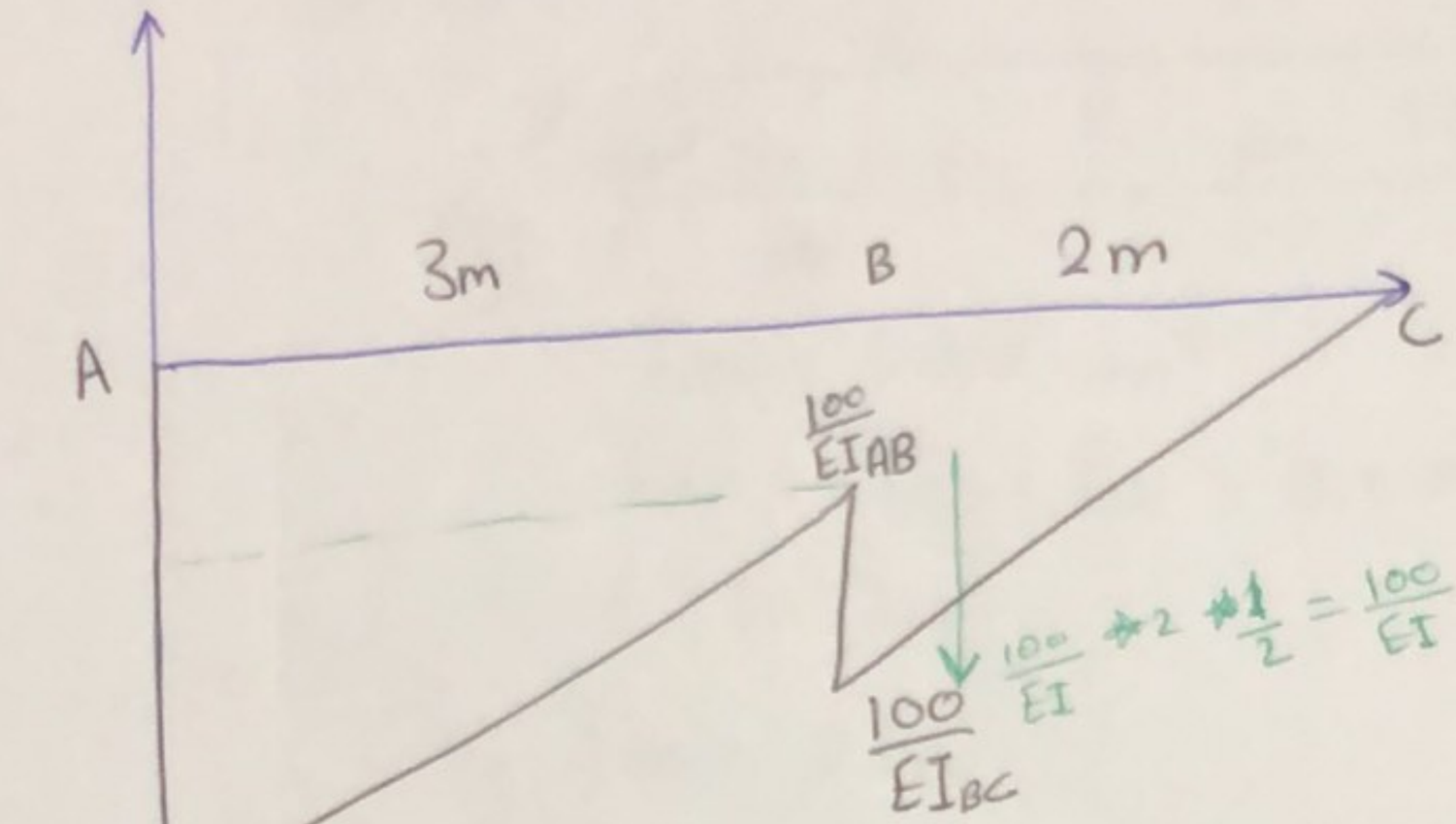
$E = 200 \ GPa$

Find : Δ_C / Δ_B / θ_B

$\Sigma Fy = 0$: $Ay = 50 \ kN$

$M_A = 250 \ KN.m$





$100/EI * 2 * 1/2 = 100/EI$

$$\Delta C = t_{C/A} = \frac{100}{EI_{BC}} * \frac{4}{3} + \frac{150}{EI_{AB}} * \frac{1}{2} * 3 * 4 + \frac{100}{EI_{AB}} * 3 * 3.5$$

$$= \frac{133}{EI_{BC}} + \frac{1950}{EI_{AB}}$$

$$= \frac{133.33}{200 * 10^6 * 3 * 10^{-4}} + \frac{1950}{200 * 10^6 * 5 * 10^{-4}}$$

$$= 0.0217 \text{ m}$$

$$\Delta_B = t_{B/A} = \frac{150}{EI_{AB}} * \frac{1}{2} * 3 * 2 + \frac{100}{EI_{AB}} * 3 * 1.5$$

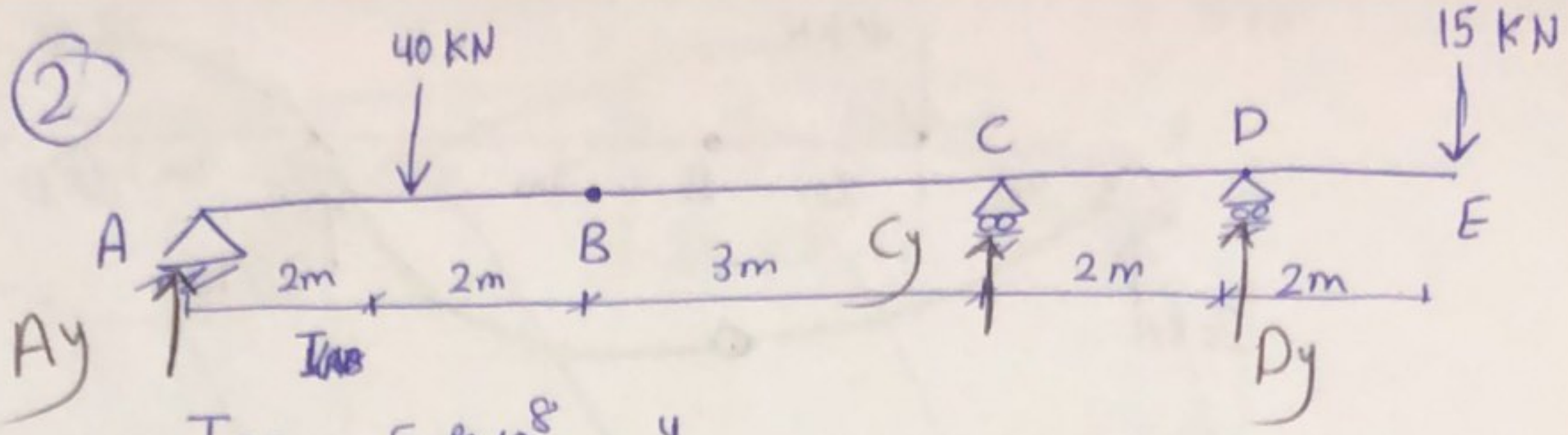
$$= \frac{900}{200 * 10^6 * 5 * 10^{-4}}$$

$$= 9 * 10^{-3} \text{ m}$$

$$\theta_B = -\theta_{B/A} = \frac{150}{EI_{AB}} * 3 * \frac{1}{2} + \frac{100 * 3}{EI_{AB}}$$

$$= 5.25 * 10^{-3} \text{ rad}$$

Q2: (B) (2)



$I_{AB} = 5 * 10^8 \ mm^4$

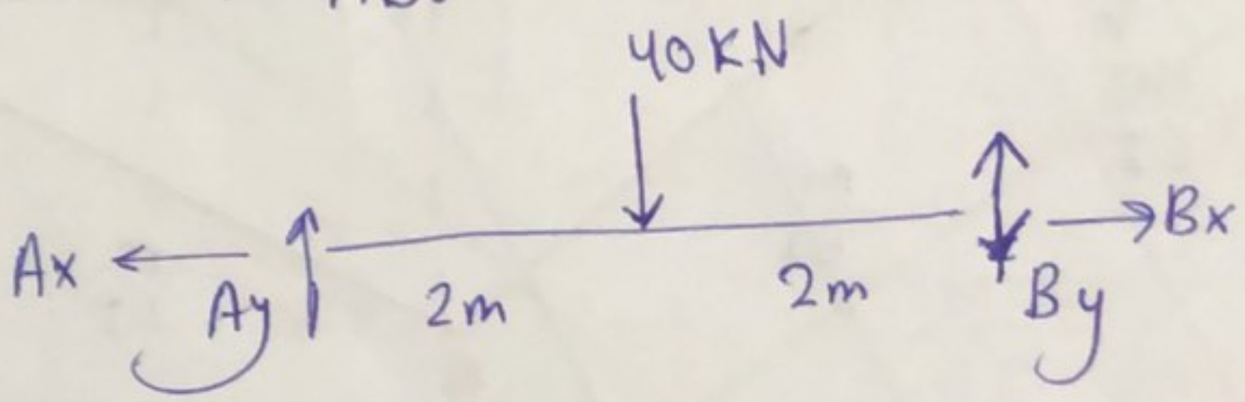
$I_{BC} = I_{CD} = I_{DE} = 2.5 * 10^8 \ mm^4$

$E = 70 \ GPa$

$I_{AB} = 2 I_{BC}$

find: $\Delta_B / \Delta_E / \theta_A / \theta_C / \theta_D / \theta_E$?

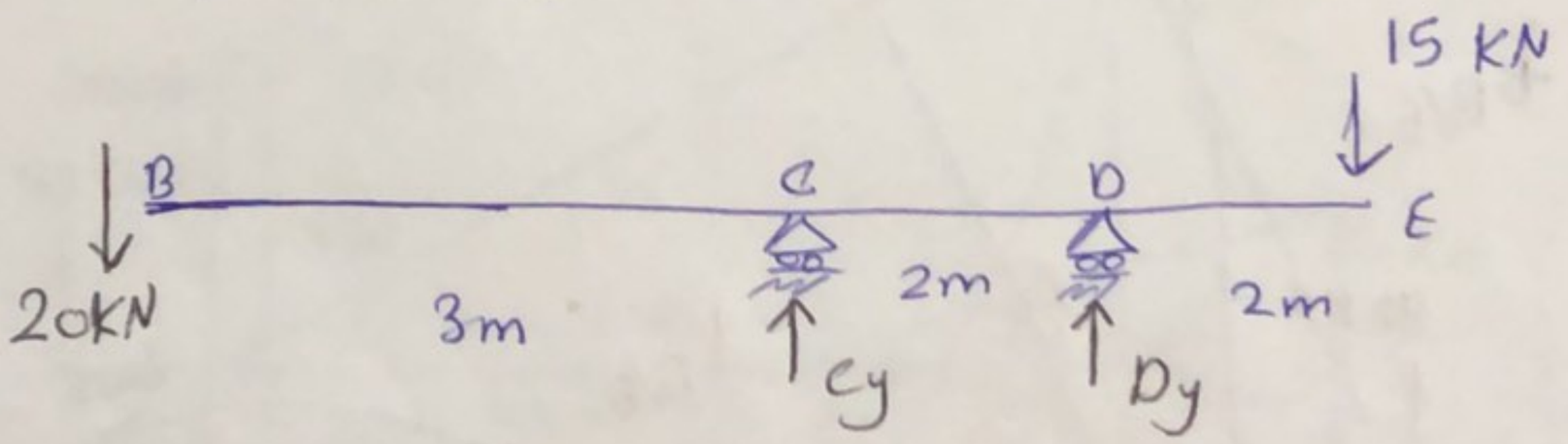
member AB:



$A_y = 20 \ KN = B_y$ "Because of Symmetry"

$A_x = B_x = 0$

member BE

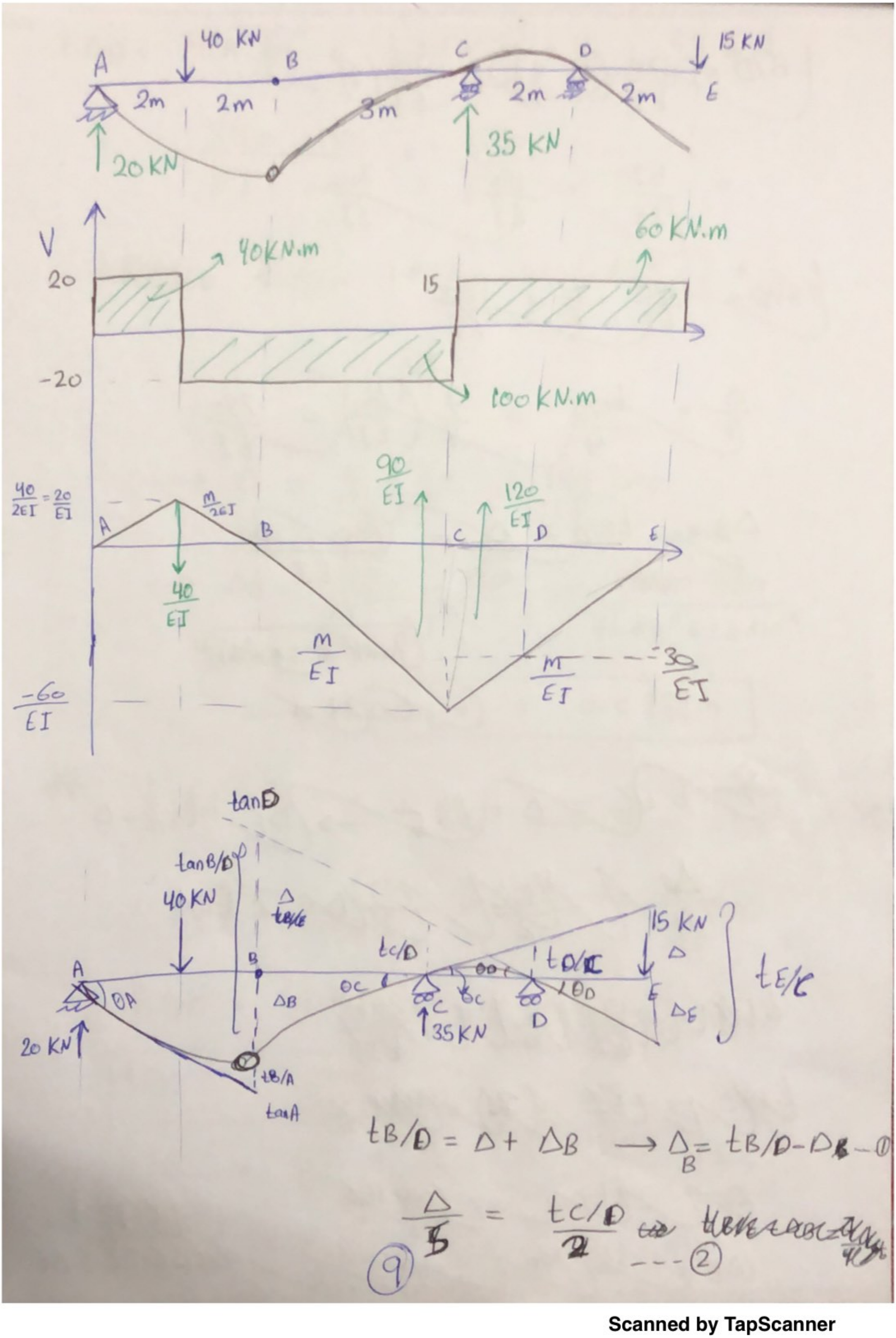


$\Sigma M_C = 0 : \quad 60 + 2D_y - 80 = 0$

$D_y = 0$

$\Sigma F_y = 0 : \quad -20 + C_y - 15 = 0$

(8) $C_y = 35 \ KN$



$$t_{B/D} = \Delta + \Delta_B \longrightarrow \Delta_B = t_{B/D} - \Delta_B \quad \text{---} (1)$$

$$\frac{\Delta}{5} = \frac{t_{C/D}}{2} \quad \text{---} (2)$$

(9)

$$t_{B/D} = 4 * \frac{60}{EI} + \frac{1}{2} * 2 * \frac{30}{EI} * \frac{11}{3} + \frac{90}{EI} * 2$$

$$= \frac{350 + 180}{EI} = \frac{530}{EI}$$

$$t_{C/D} = 1 * \frac{60}{EI} + \frac{1}{2} * 2 * \frac{30}{EI} * \frac{2}{3}$$

$$= \frac{80}{EI}$$

$$\longrightarrow \Delta = \frac{5 * 80}{2\,EI} = \frac{140}{EI}\ 200$$

$$\longrightarrow \Delta_B = \frac{530 - 140}{EI}\ ^{200} = \frac{330}{70 * 10^6 * 2.5 * 10^{-4}}$$

$$\longrightarrow \boxed{\Delta_B = 0.0189\,m}$$

$$* \quad t_{E/C} = \Delta + \Delta_E \longrightarrow \Delta_E = t_{E/C} - \Delta \text{ --(}$$

$$\frac{\Delta}{4} = \frac{t_{D/C}}{2} \longrightarrow \Delta = 2\,t_{D/C}$$

$$t_{D/C} = 1 * \frac{60}{EI} + \frac{1}{2} * 2 * \frac{30}{EI}\left(2 - \frac{2}{3}\right)$$

$$\boxed{t_{D/C} = \frac{100}{EI}}$$

$$t_{E/C} = \frac{120}{EI} * \frac{8}{3} = \boxed{\frac{320}{EI} = t_{E/C}}$$

$$\Delta = 2 * \frac{100}{EI} = \boxed{\frac{200}{EI} = \Delta}$$ (10)

$$\Delta_E = \frac{320 - 200}{EI}$$

$$= 6.86 * 10^{-3} \ m$$

$$\theta_D = \tan^{-1}\left(\frac{t_{C/D}}{2}\right) = \tan^{-1}\left(\frac{80}{2EI}\right)$$

$$= \tan^{-1}\left(2.286 * 10^{-3}\right)$$

$$= 0.131 \text{ rad}$$

$$\theta_C = \tan^{-1}\left(\frac{t_{D/C}}{2}\right) = \tan^{-1}\left(\frac{100}{2EI}\right)$$

$$= \tan^{-1}\left(2.857 * 10^{-3}\right)$$

$$= 0.164 \text{ rad}$$

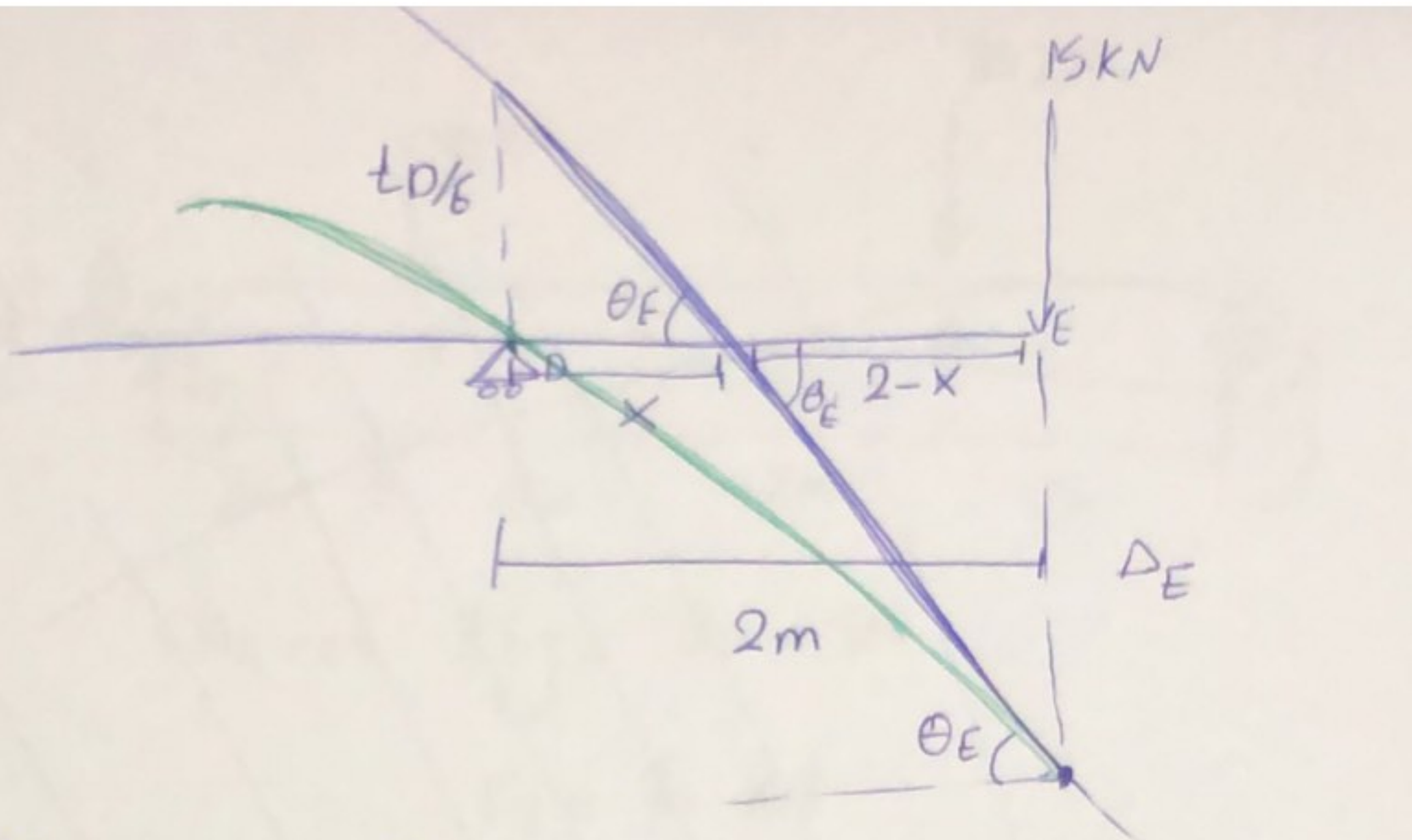
$$\theta_A = \tan^{-1}\left(\frac{\Delta_B + t_{B/A}}{4}\right)$$

$$\Delta_B = 0.022 \ m$$

$$t_{B/A} = \frac{40}{EI} * 2 = \frac{80}{EI} = 4.57 * 10^{-3}$$

$$\theta_A = \tan^{-1}\left(\frac{0.0189}{4}\right) = 0.36^\circ$$

$$= 5.85 * 10^{-3} \text{ rad}$$



$$\frac{x}{t_{D/E}} = \frac{2-x}{\Delta_E}$$

$$\frac{x}{2-x} = \frac{t_{D/E}}{6.86*10^{-3}} = \frac{\frac{1}{2}*2*\frac{10}{EI}*\frac{2}{3}}{6.86*10^{-3}}$$

$$\rightarrow \quad \frac{x}{2-x} = \frac{20}{EI*6.86*10^{-3}}$$

$$\frac{x}{2-x} = 0.1666$$

$$6.0025\,x = 2-x$$

$$x = 0.286\ m$$

$$\tan\theta_E = \frac{t_{D/E}}{x}$$

$$\theta_E = \tan^{-1}\left(\frac{1.143*10^{-3}}{0.286}\right)$$

$$\theta_E = 0.229^\circ = 3.99*10^{-3}\ rad$$

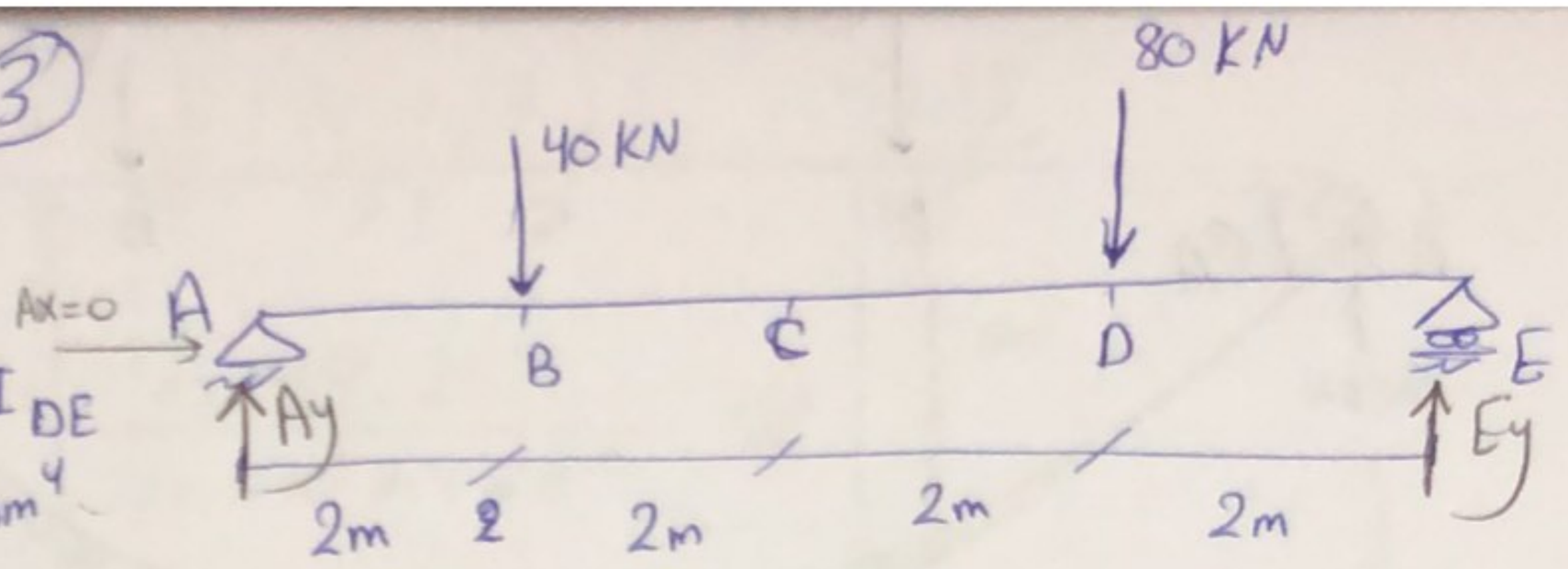
Q2: (3)

$E = 200\,GPa$

$I_{BD} = 2I_{AB} = 2I_{DE}$
$= 8 * 10^8\ mm^4$

Δ_B / D_O ?

$\Theta_A / \Theta_C / \Theta_E$?

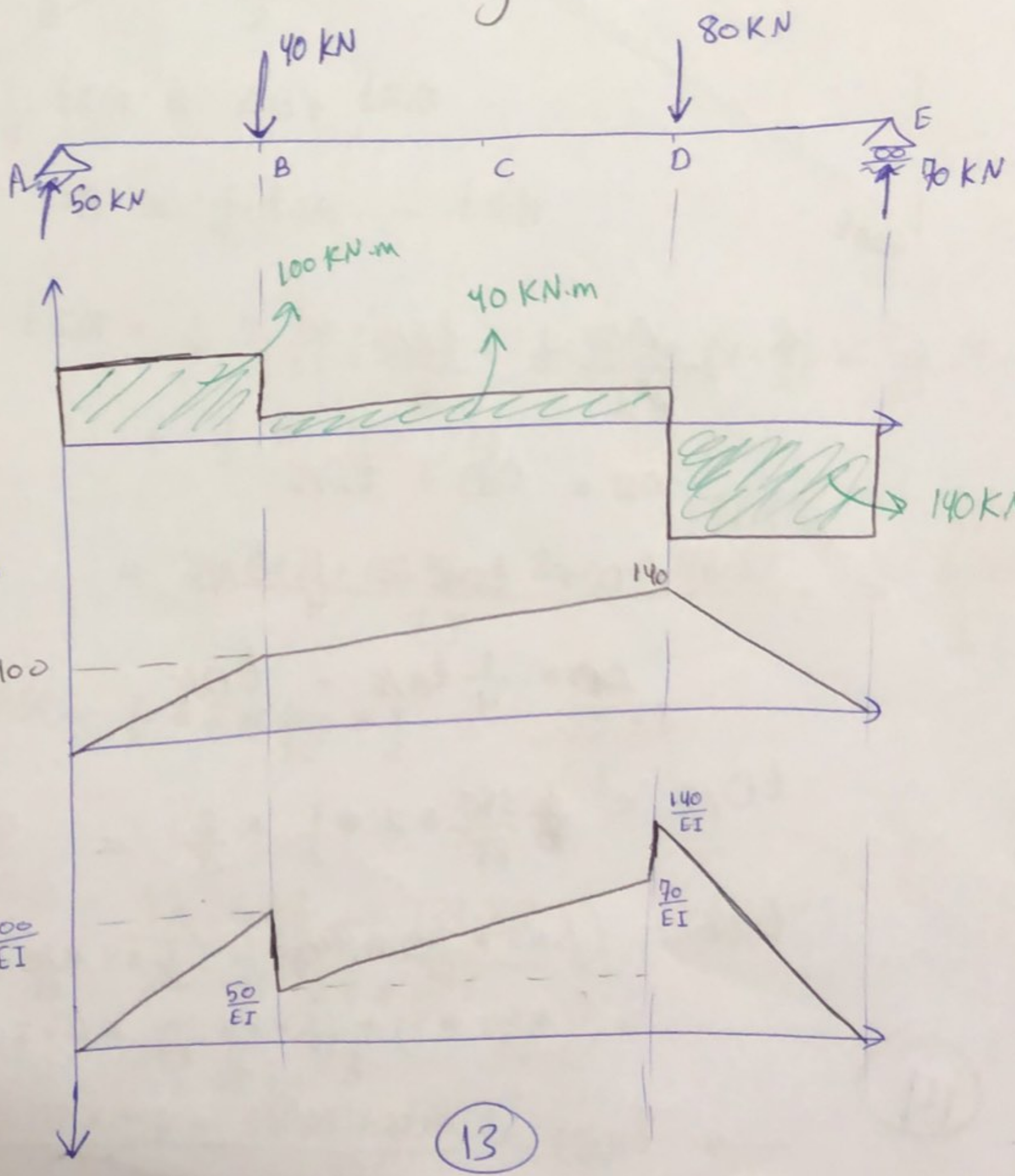


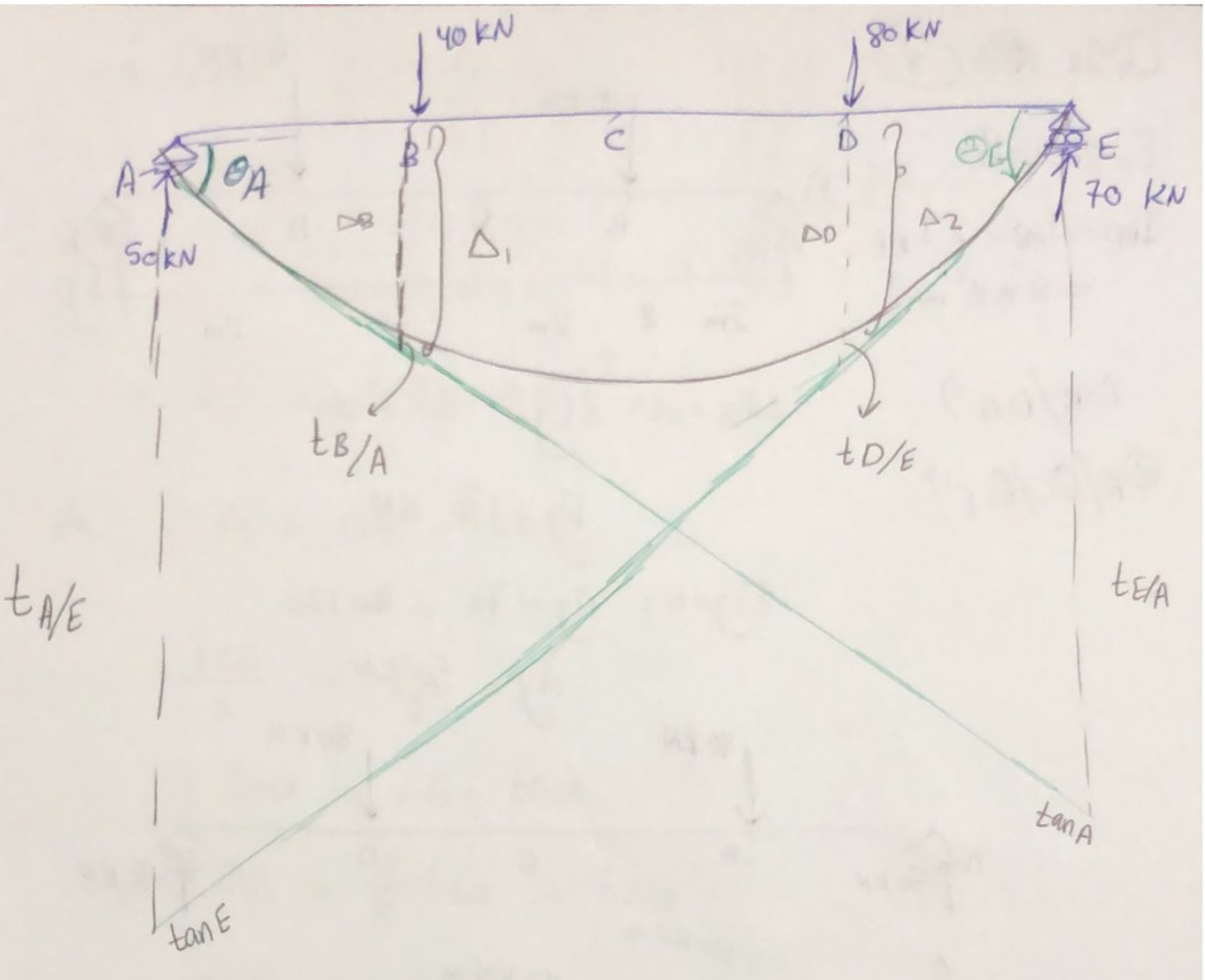
$\Sigma M_A = 0$: $8Ey = 80 + 480$

$Ey = 70\ KN$

$\Sigma Fy = 0$: $Ay + 70 = 120$

$Ay = 50\ KN$





$$\frac{\Delta 2}{\cancel{\delta} 2} = \frac{t_{A/E}}{8}$$

$$\Delta 2 = \Delta D + t_{D/E}$$

$$\longrightarrow \Delta D + t_{D/E} = \frac{1}{4} t_{A/E}$$

$$\Delta D = \frac{1}{4} t_{A/E} - t_{D/E}$$

$$t_{D/E} = \cancel{6} \frac{140}{EI} * 2 * \frac{1}{2} * \frac{2}{3} = \frac{93.33}{EI}$$

$$t_{A/E} = \left(\frac{1}{2} * 2 * \frac{100}{EI} * \left(\cancel{6} \frac{4}{3}\right)\right) + \left(\frac{1}{2} * 4 * \frac{200}{EI}\left(2 + \frac{8}{3}\right)\right)$$

$$+ \left(4 * \frac{50}{EI} * 4\right) + \left(\frac{1}{2} * 2 * \frac{140}{EI} * \left(6 + \frac{2}{3}\right)\right)$$

$$\longrightarrow t_{A/E} = \frac{133.33 + 186.67 + 800 + 933.33}{EI} = \frac{2050.33}{EI}$$

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$$\rightarrow \Delta_D = \frac{1}{4}\left(\frac{2050.33}{EI}\right) - \frac{93.33}{EI}$$

$$= \frac{419.2525}{200 * 10^6 * 4 * 10^{-4}} =$$

$$\rightarrow \Delta_D = 5.24 * 10^{-3} \text{ m}$$

$$* \quad \Delta_1 = \Delta_B + t_{B/A}$$

$$\frac{t_{E/A}}{8} = \frac{\Delta_1}{2}$$

$$\frac{1}{4} t_{E/A} = \Delta_B + t_{B/A}$$

$$\Delta_B = \frac{1}{4} t_{E/A} - t_{B/A}$$

$$t_{E/A} = \frac{1}{2} \cdot 2 \cdot \frac{100}{EI}\left(6 + \frac{2}{3}\right) + \frac{1}{2} \cdot 4 \cdot \frac{20}{EI}\left(2 + \frac{4}{3}\right) + \frac{50}{EI} \cdot 4 \cdot 4$$

$$+ \frac{1}{2} \cdot 2 \cdot \frac{140}{EI} \cdot \frac{4}{3}$$

$$= \frac{666.67 + 133.33 + 800 + 186.67}{EI} = \frac{1786.67}{EI}$$

$$t_{B/A} = \frac{1}{2} * 2 * \frac{100}{EI} * \frac{4}{3} = \frac{133.33}{EI}$$

$$\Delta_B = \frac{446.675 - 133.33}{200 * 10^6 * 4 * 10^{-4}}$$

$$\rightarrow \Delta_B = 3.9 * 10^{-3} \text{ m}$$

(15)

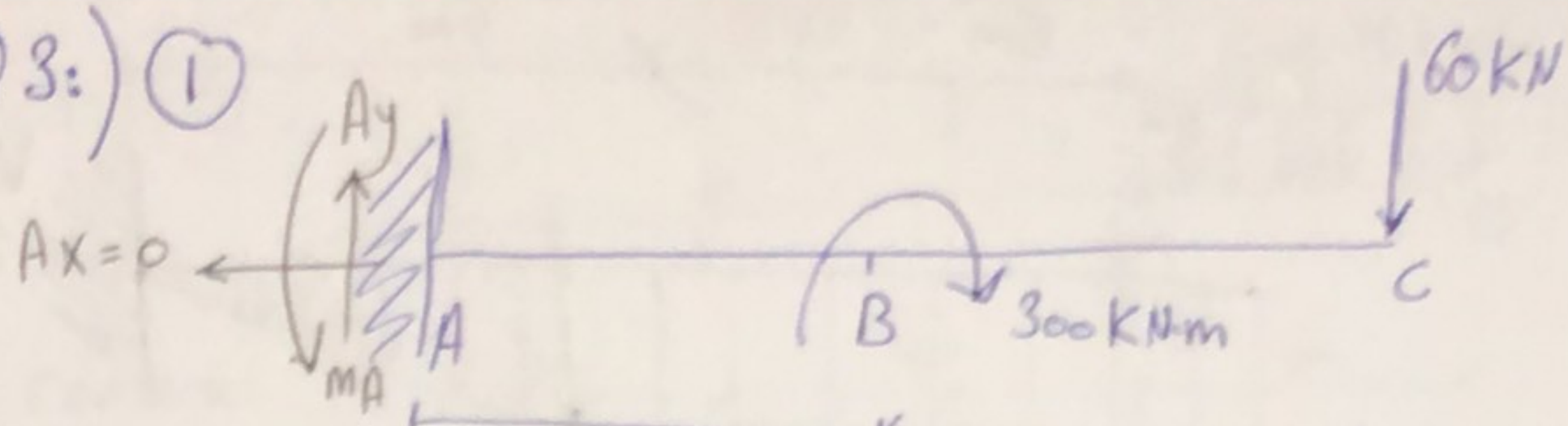
$$\theta_A = \tan^{-1}\left(\frac{t_{E/A}}{8}\right) = \tan^{-1}\left(\frac{1786.67}{8EI}\right)$$

$$\longrightarrow \theta_A = 2.79 * 10^{-3} \text{ rad}$$

$$\theta_E = \tan^{-1}\left(\frac{t_{A/E}}{8}\right) = \tan^{-1}\left(\frac{2050.33}{8EI}\right)$$

$$\longrightarrow \theta_E = 3.20 * 10^{-3} \text{ rad}$$

Q3:) (1)



EI = Constant

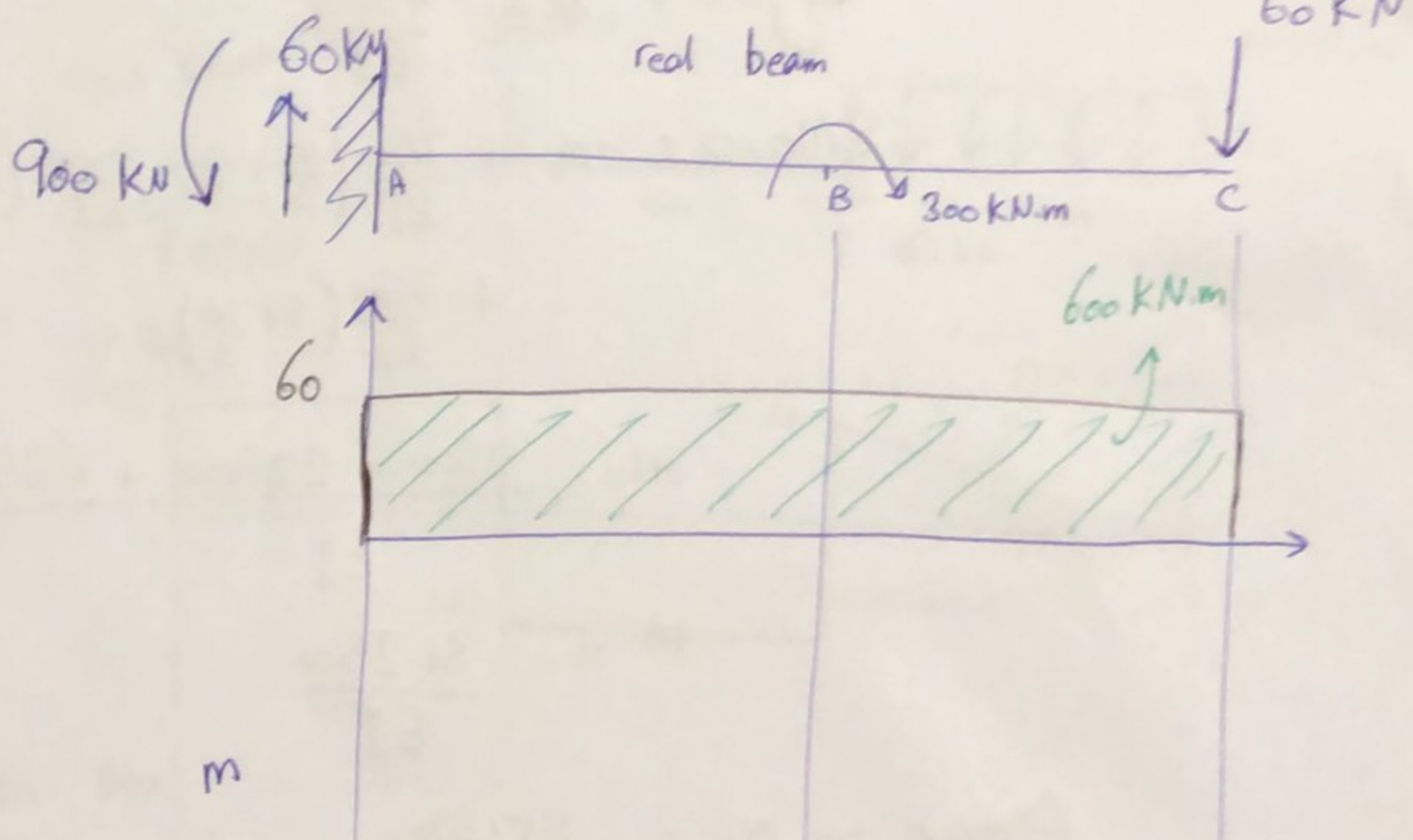
E = 200 GPa

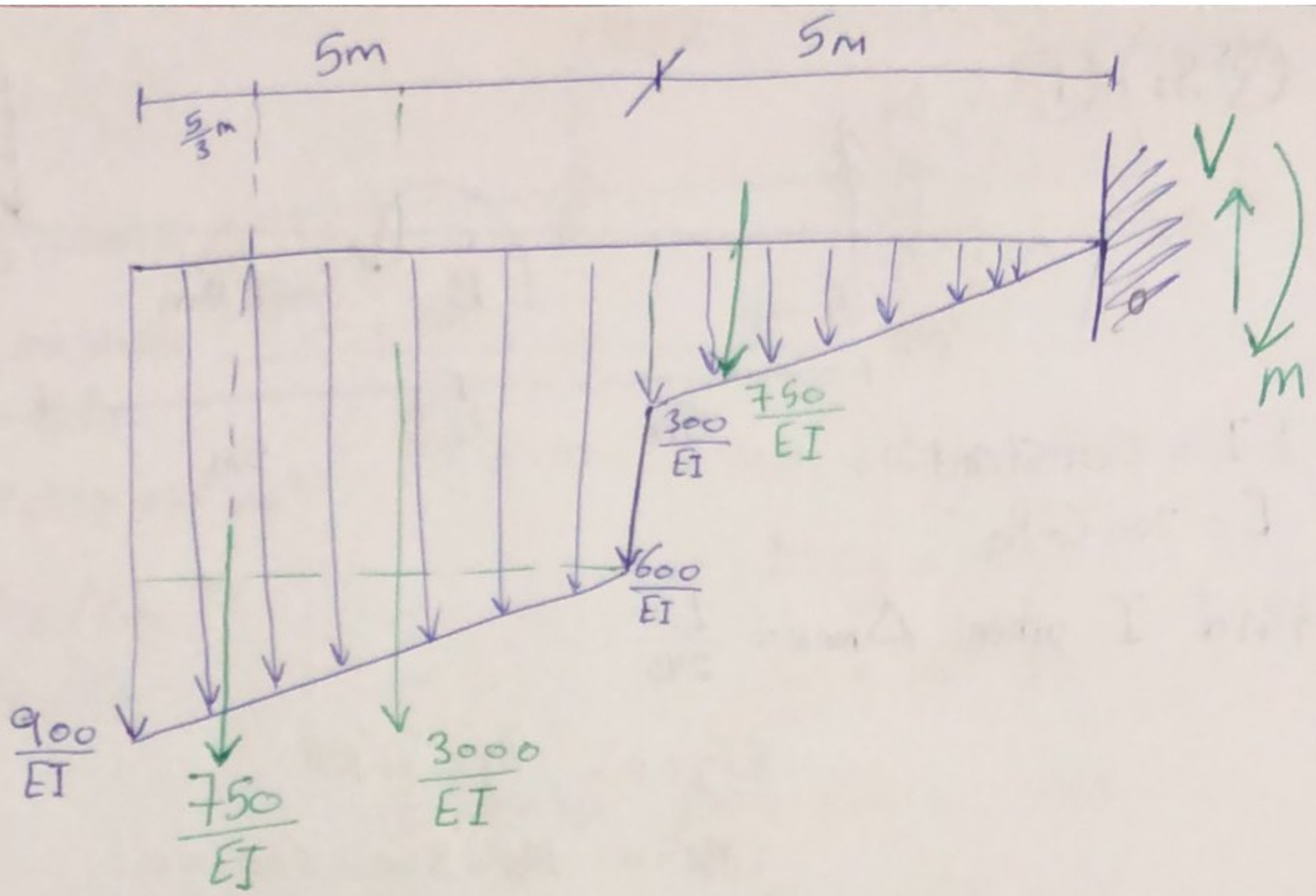
find I when $\Delta_{max} = \frac{L}{240}$.

$\Sigma F_y = 0$: $A_y = 60\ kN$

$\Sigma M_A = 0$: $M_A - 300 - 600 = 0$

$M_A = 900\ kN.m$





$$\Sigma F_y = 0 : \quad V = \frac{4500}{EI}$$

$$\Sigma M_O = 0 : \quad -m + \frac{750}{EI}\left(\frac{10}{3}\right) + \frac{3000}{EI}\left(\frac{15}{2}\right) + \frac{750}{EI}\left(5 + \frac{10}{3}\right) = 0$$

$$m = \frac{2500 + 22500 + 6250}{EI}$$

$$m = \frac{31250}{EI}$$

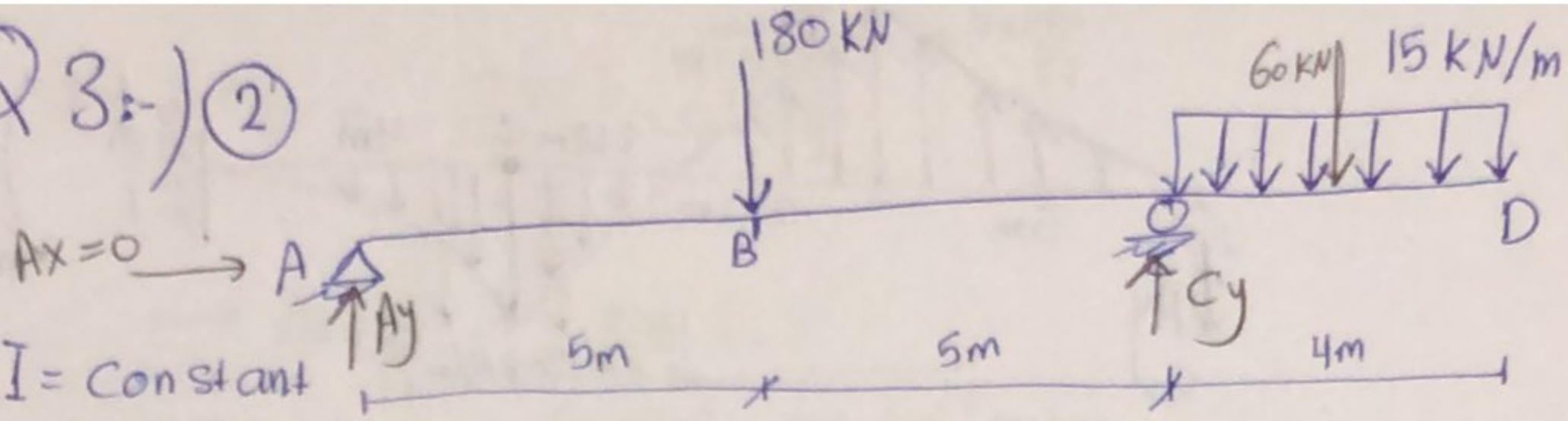
$$\Delta_{max} = m = \frac{31250}{EI} = \frac{L}{240}$$

$$I = \frac{31250 * 240}{200 * 10^6 * 10}$$

$$I = 3.75 * 10^{-3}\ m^4$$

$$= 37.5 * 10^{8}\ mm^4$$

Q 3:-) ②



EI = Constant

E = 70 GPa

I = 2.340 * 10^6 mm^4

Δ_B / Δ_D ?

θ_A / θ_C ?

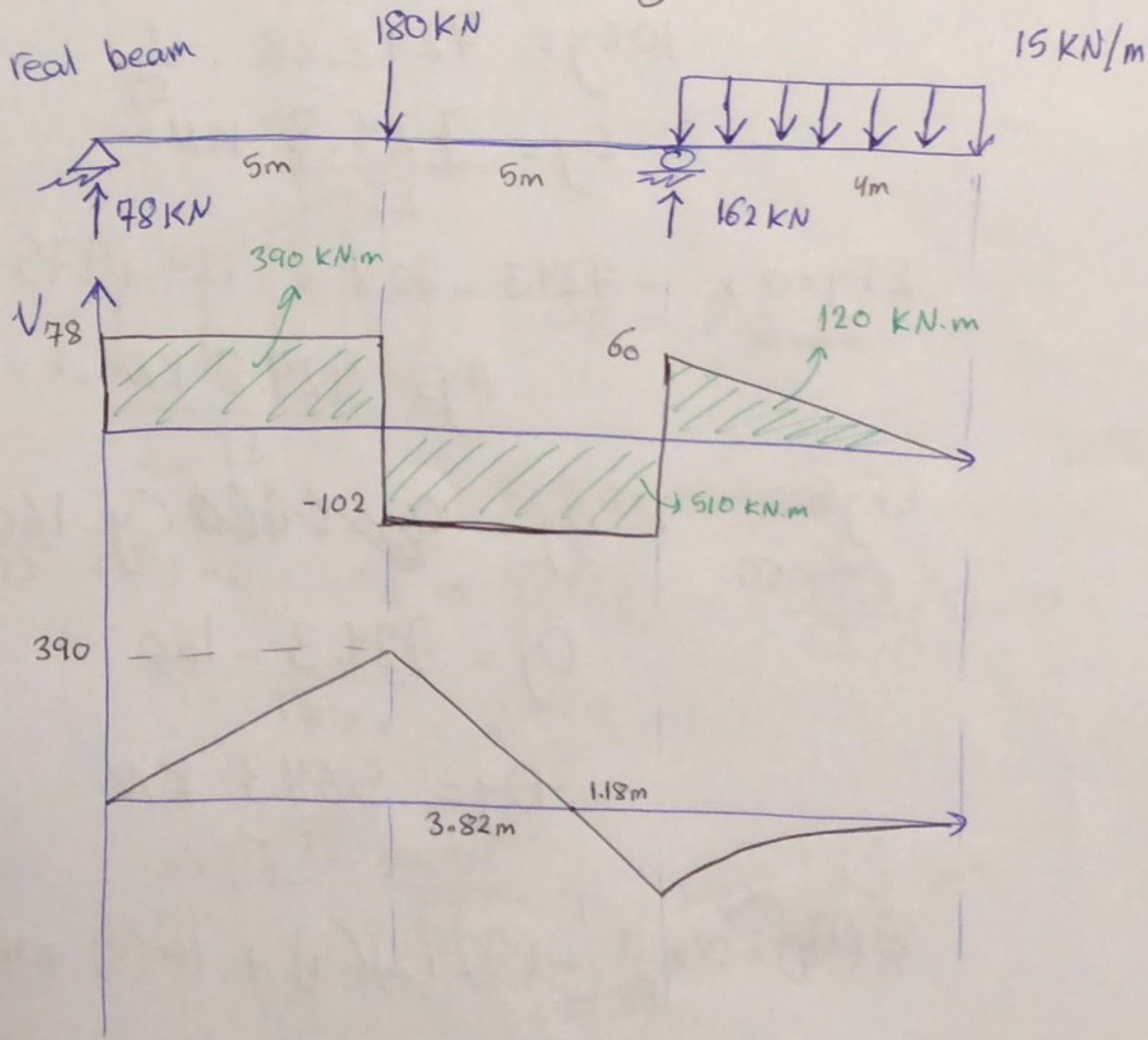
$\Sigma M_{A=0}$: $-900 + 10Cy - 720 = 0$

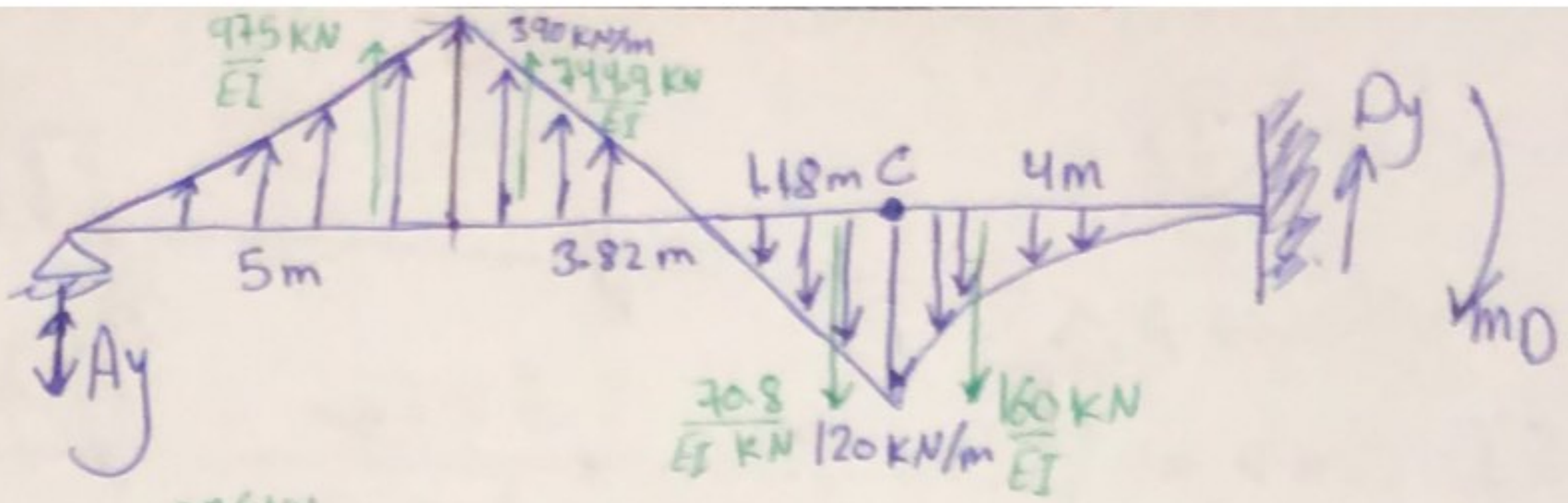
$10Cy = 1620$

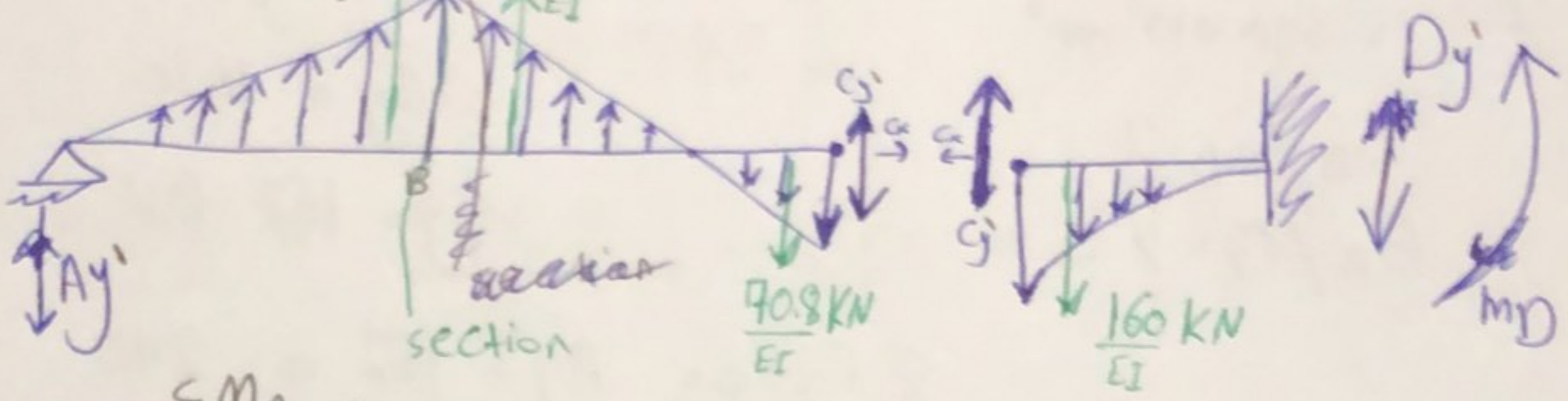
$Cy = 162$ kN

$\Sigma Fy=0$: $Ay + 162 = 240$

$Ay = 78$ kN







$$\Sigma M_A = 0 : -10\,C_y - 680.125 + 4673.006 + 3250 = 0$$

$$10\,C_y = 7242.88 \cdot \frac{1}{EI}$$

$$C_y = \frac{724.3}{EI}\ KN$$

$$\Sigma F_y = 0 \text{ (member Ac)} : -724.3 - 70.8 + 744.9 + 975 - A_y = 0$$

$$A_y = \frac{924.8\ KN}{EI} \cdot$$

$$\Sigma F_y = 0 \text{ (member CD)} : D_y = C_y - \frac{160}{EI}$$

$$D_y = \frac{724.3 - 160}{EI}$$

$$D_y = \frac{564.3\ KN}{EI}$$

$$\Sigma M_D = 0 : \frac{1}{EI}\left[-(724.3)(4) + 160(3) + m_D\right] = 0$$

$$m_D = \frac{2417.2\ KN.m}{EI}$$

(20)

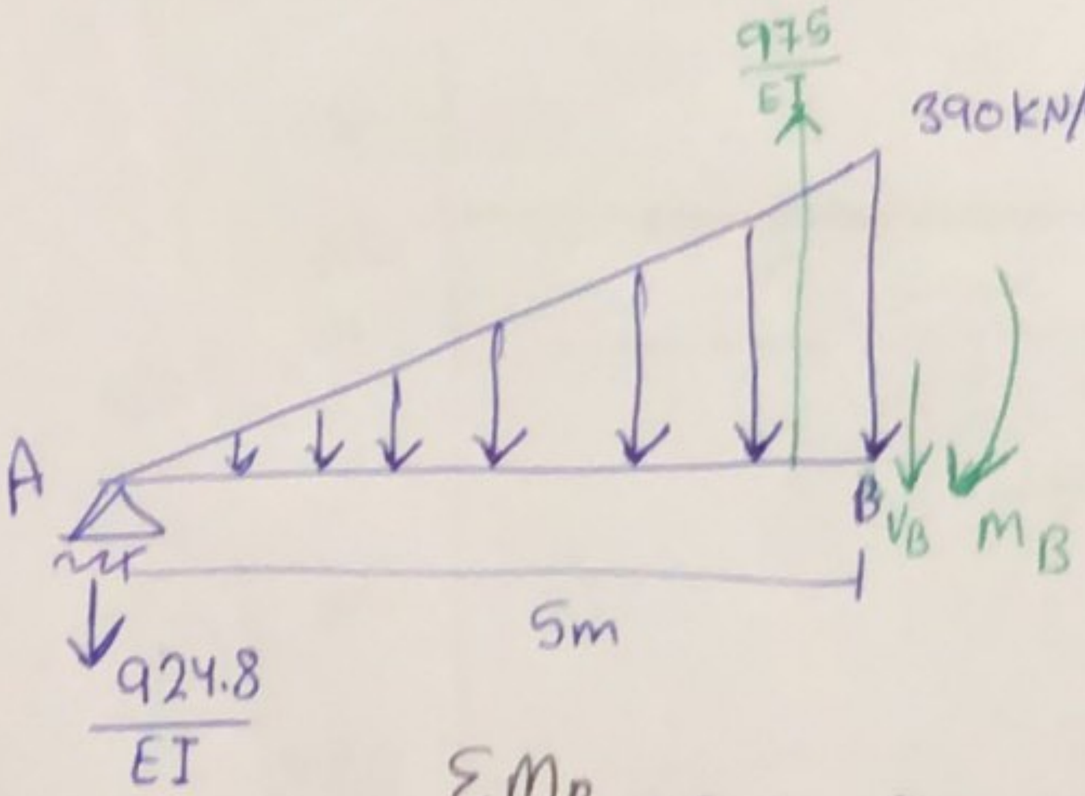
$$\Delta_D = m_D = \frac{2417.2}{EI}$$

$$\rightarrow \Delta_D = \frac{2417.2}{70*10^6 * 2340*10^{-6}}$$

$$= 0.01476 \text{ m}$$

$$\theta_A = Ay = \frac{924.8}{EI} = -5.65 *10^{-3} \text{ rad}$$

$$\theta_C = Cy = \frac{724.3}{EI} = -4.42 *10^{-3} \text{ rad}$$



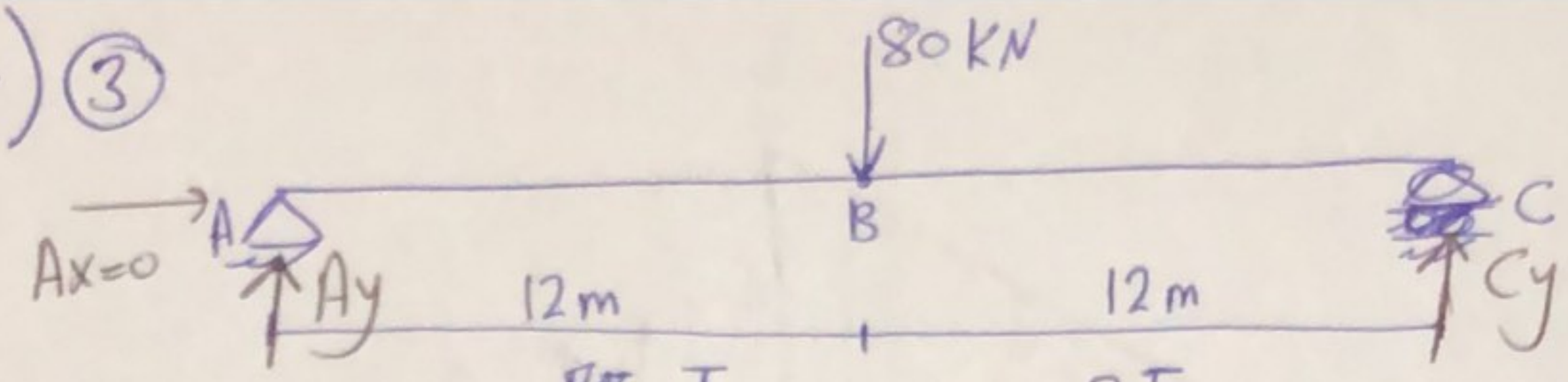
$$\Delta_B = m_B$$

$$\Sigma M_B = 0 : -m_B + \frac{975}{EI}\left(\frac{5}{3}\right) + \frac{924.8}{EI}(5) = 0$$

$$m_B = \frac{2999}{EI}$$

$$\Delta_B = \frac{2999}{EI} = 0.0183 \text{ m}$$

Q3:-) ③



E = Constant = 200 GPa

$I = 600(10^6)\ mm^4$

Δ_B ?

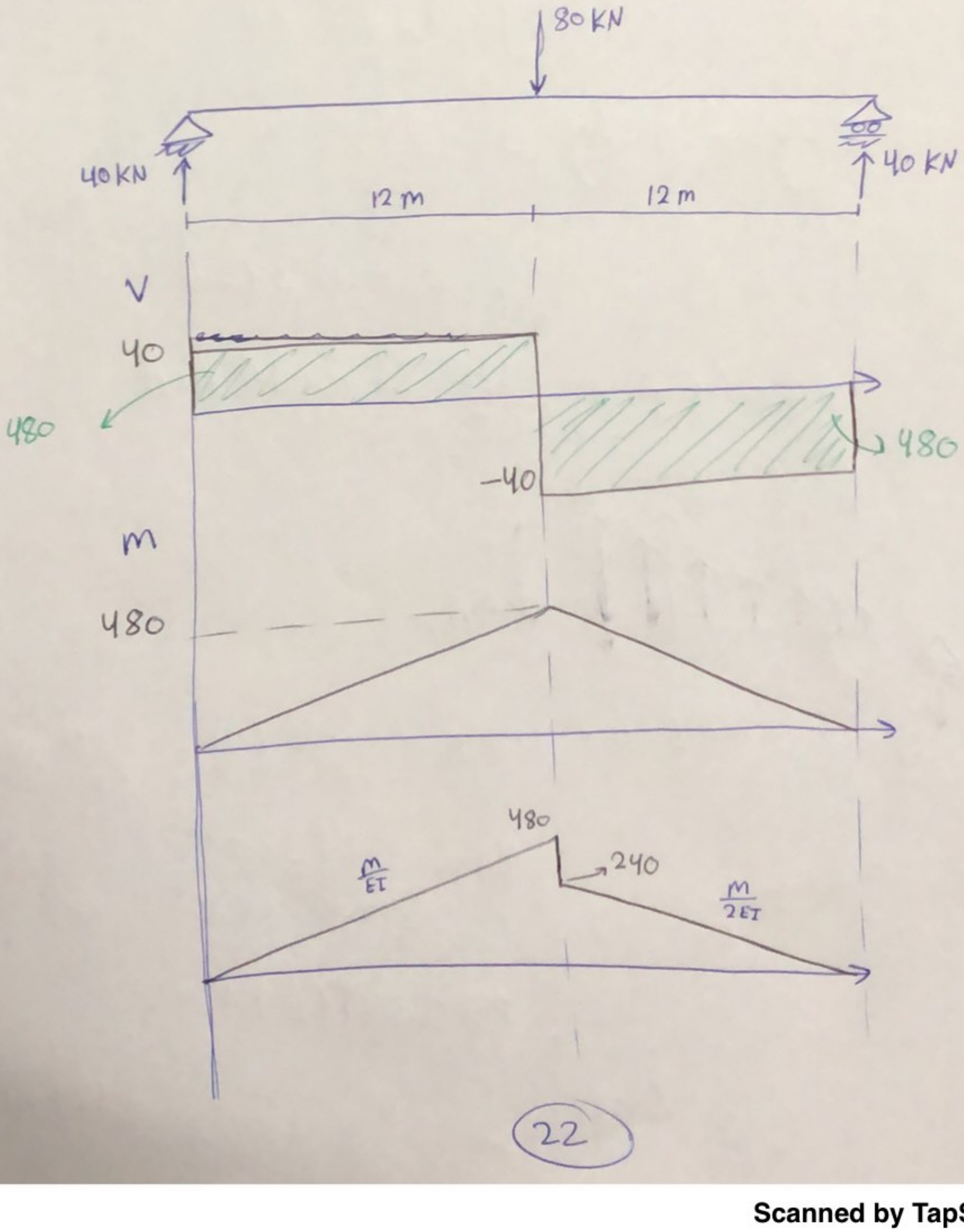
$\Sigma M_A = 0: \quad 24\,Cy = 960$

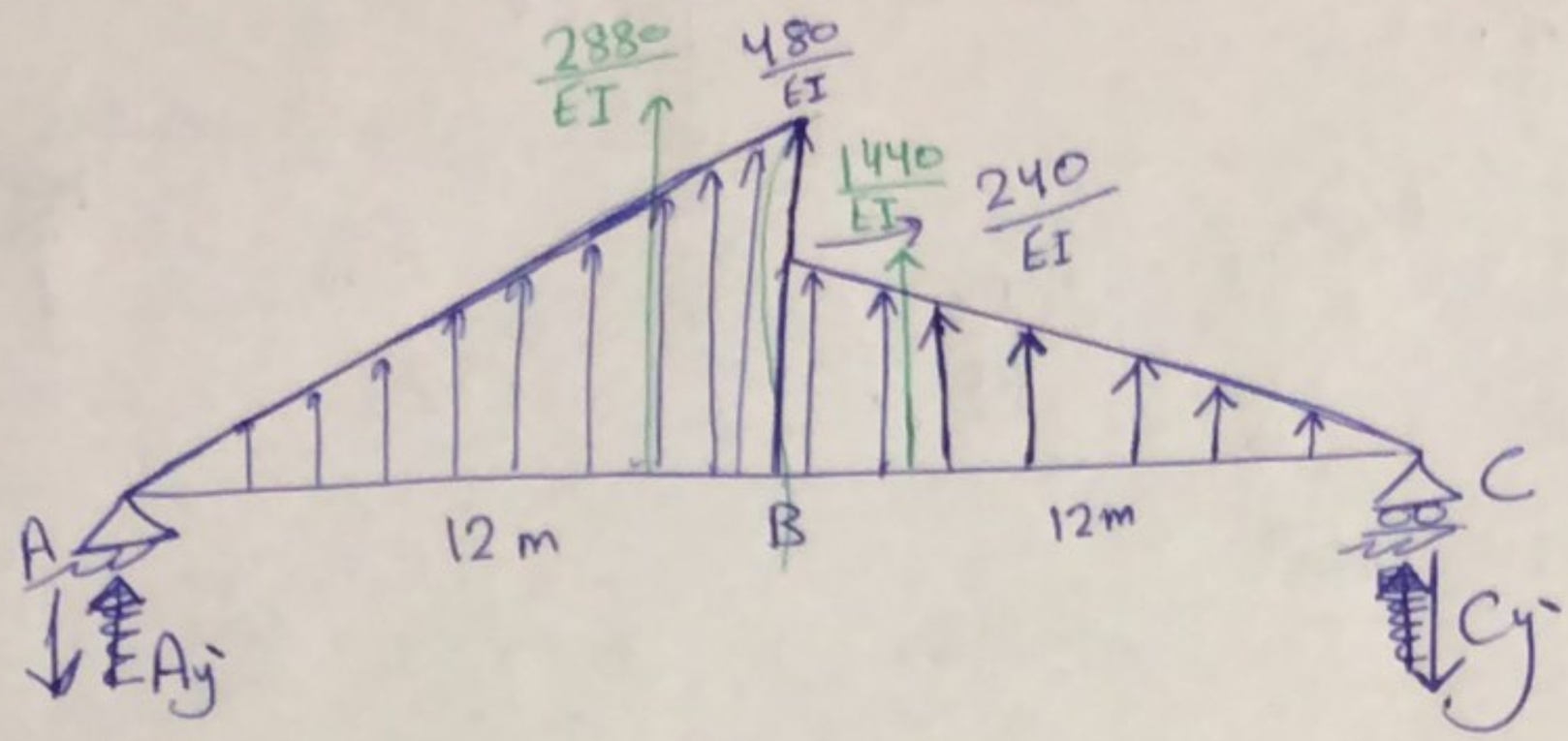
θ_A ?

$Cy = 40\ kN$

θ_C ?

$\Sigma Fy = 0: \quad Ay = 40\ kN$





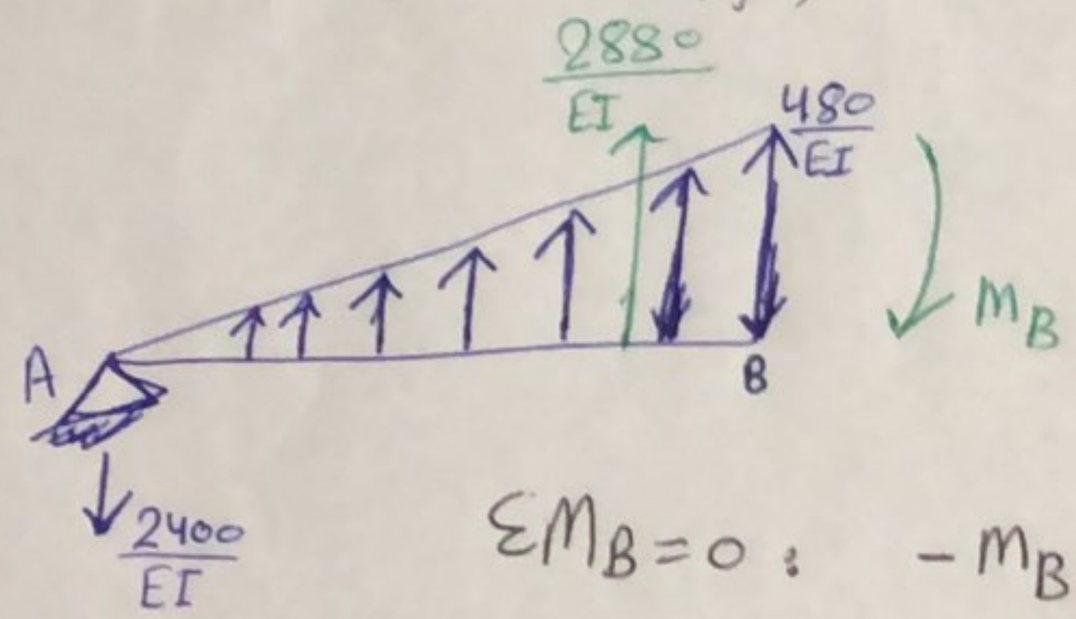
$$\Sigma M_A = 0 : \quad -24\,Cy + \frac{1440}{EI}(16) + \frac{2880}{EI}(8) = 0$$

$$Cy = \frac{1920}{EI} \text{ KN}$$

$$\Sigma F_y = 0 : \quad Ay = \frac{2400}{EI} \text{ KN}$$

$$\Theta_{A\,(real)} = Ay_{(conjugate)} = \frac{2400}{EI} = 0.02 \text{ rad}$$

$$\Theta_{C\,(real)} = Cy_{(conjugate)} = \frac{1920}{EI} = 0.016 \text{ rad}$$



$$\Sigma M_B = 0 : \quad -M_B - \frac{2880}{EI}(4) + \frac{2400}{EI}(12) = 0$$

$$M_B = \frac{17280}{200.10^6 . 600.10^{-6}}$$

$$MB = 0.144 \text{ KN.m}$$

(23) $\Delta B_{(real)} = M_{B(conjugate)} = 0.144 \text{ m}$